Comment on "Quantum Strategy Without Entanglement"

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We make remarks on the paper of Du et al (quant-ph/0011078) by pointing out that the quantum strategy proposed by the paper is trivial to the card game and proposing a simple classical strategy to make the game in classical sense fair too.

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In the paper of Du et al [1], they present a two players' game just as the following card game. There are three cards, otherwise identical, except for the following markings: the first card has a circle on each side; the second card has a dot on each side; the third card has a circle on one side and a dot on the other. Alice puts the three cards in a black box and shakes it to randomize the three cards. Bob is allowed to draw one card out from the box without seeing the cards. If the card has the same marks on both sides, Alice wins one. Otherwise, Bob wins one. Obviously, this game is unfair to Bob because that the possibility to draw out an identical face card in three cards is $\frac{2}{3}$, and Alice has the expected payoff $\bar{\pi}_A = \frac{2}{3} \times 1 + \frac{1}{3} \times (-1) = \frac{1}{3}$ while Bob has the expected payoff $\bar{\pi}_B = \frac{1}{3} \times 1 + \frac{2}{3} \times (-1) = -\frac{1}{3}$ [2]. Actually, if Bob see all the upper faces of the three cards before drawing, he will make sure that the card with different upper face in three cards must be a card with the identical faces, so he will randomly draw one of the two other cards, which make him win the game with a fifty-fifty chance. But this observation is forbidden.

Du et al propose a quantum strategy to make this game fair by adding two principles such as:

1. Allow Bob to make a single *query* by calling a quantum oracle to the black box;

2. Allow Bob to withdraw from the game once he knows the upper face of the card he draws is different to the upper faces of two other cards.

It is obvious that only adding the principle 2 to the card game helps Bob nothing because that he isn't able to make such a decision without any other information. So the principle 1 is necessary. However, we will show that the quantum oracle proposed by Du et al is trivial.

Considering the quantum oracle, let the card's state be $|0\rangle$ if the upper face is a circle or $|1\rangle$ if the upper face is a dot. So the three-card state is $|r\rangle = |r_0 r_1 r_2\rangle$, where
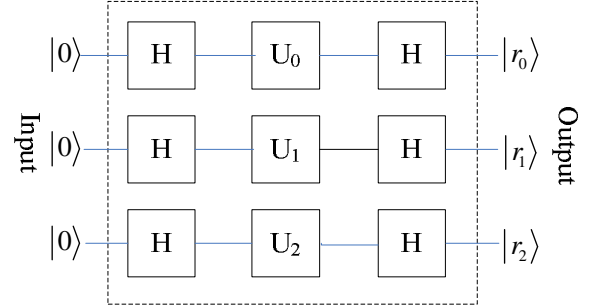
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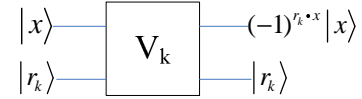
FIG. 1: The proposed quantum oracle for card game

FIG. 2: The equivalent quantum circuit of U_k

$r_k \in \{0, 1\}$. The proposed quantum oracle in [1] is just as Fig. 1 which is little different from the original figure in their paper. Note that the inner structure of their oracle is shown here.

In Fig. 1, as part of the quantum oracle, the following unitary matrix is required:

$$U_k = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\pi r_k} \end{pmatrix} = \begin{cases} I_2, & r_k = 0; \\ \sigma_z, & r_k = 1. \end{cases} \tag{1}$$

If we want to construct the oracle we should know all values of $r_k (k = 0, 1, 2)$. Actually, U_k is just the quantum circuit shown in Fig. 2, i.e. $V_k|x\rangle|r_k\rangle = (-1)^{r_k \cdot x}|x\rangle|r_k\rangle$. So the original oracle is equivalent to be added one input, i.e. $|r_k\rangle$, for each $|0\rangle$. The transformation is as follows.

$$
\begin{aligned}
&(H \otimes I)V_k(H \otimes I)|0\rangle|r_k\rangle \\
=\ &(H \otimes I)V_k(\tfrac{|0\rangle+|1\rangle}{\sqrt{2}})|r_k\rangle \\
=\ &(H \otimes I)\tfrac{1}{\sqrt{2}}\left((-1)^{r_k \cdot 0}|0\rangle + (-1)^{r_k \cdot 1}|1\rangle\right)|r_k\rangle \\
=\ &\tfrac{1}{2}\left((|0\rangle + |1\rangle) + (-1)^{r_k}(|0\rangle - |1\rangle)\right)|r_k\rangle \\
=\ &\tfrac{1}{2}\left((1 + (-1)^{r_k})|0\rangle + (1 - (-1)^{r_k})|1\rangle\right)|r_k\rangle \\
=\ &|r_k\rangle|r_k\rangle
\end{aligned}
\tag{2}
$$

Obviously this transformation is trivial to the game. So adding the principle 1 to the game is equivalent to adding a third player from which Bob could know the information about all the r_k. If Bob is allowed to know the information about r_k and the principle 2 is available, the game in classical sense is fair to Bob too.

In the classical sense, in fact, we can simply alert a principle of the game and make it fair to both. The strategy is that if Alice wins she only gets one but if Bob wins he gets two. Now the payoffs are $\bar{\pi}_A = \frac{2}{3} \times 1 + \frac{1}{3} \times (-2) = 0$, $\bar{\pi}_B = \frac{1}{3} \times 2 + \frac{2}{3} \times (-1) = 0$, and the game is a zero-sum game thus a fair game.

[1] J. Du, X. Xu, H. Li, M. Shi, X. Zhou and R. Han, quant-ph/0011078.

[2] J.O. Grabbe, quant-ph/0506219.